Sol'ns

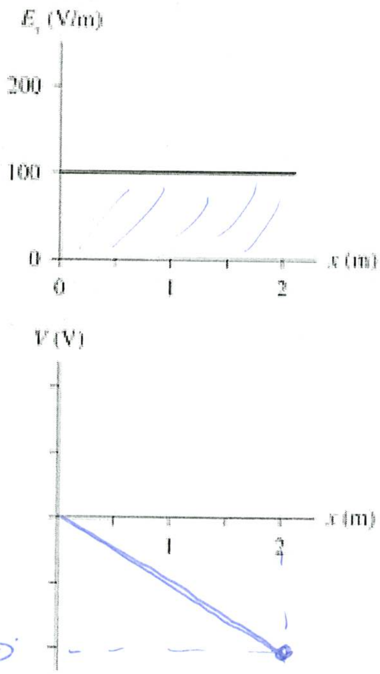
1.

The top graph shows the x-component of $\vec{E}$ as a function of x. On the axes below the graph, draw the graph of V versus x in this same region of space. Let $V = 0$ V at $x = 0$ m. Include an appropriate vertical scale. (Hint: Integration is the area under the curve.)

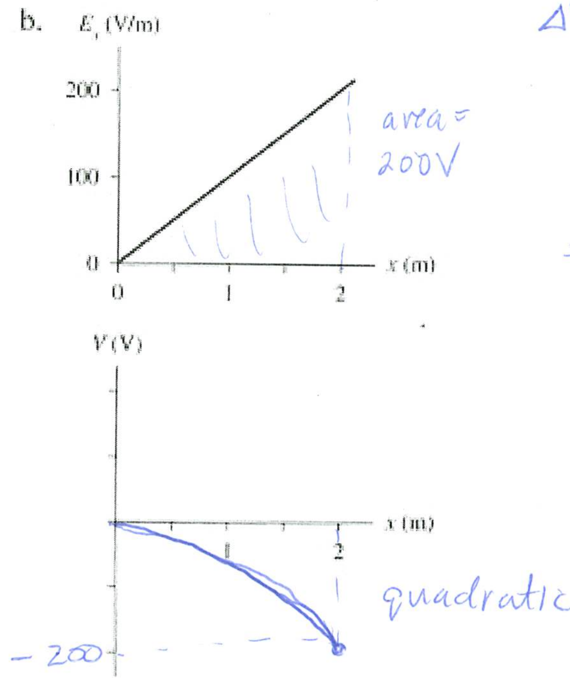
$$\Delta V = -\int \vec{E}\cdot d\vec{l} = -\int E_x\,dx = -(\text{area})$$

a.

area = 200 V

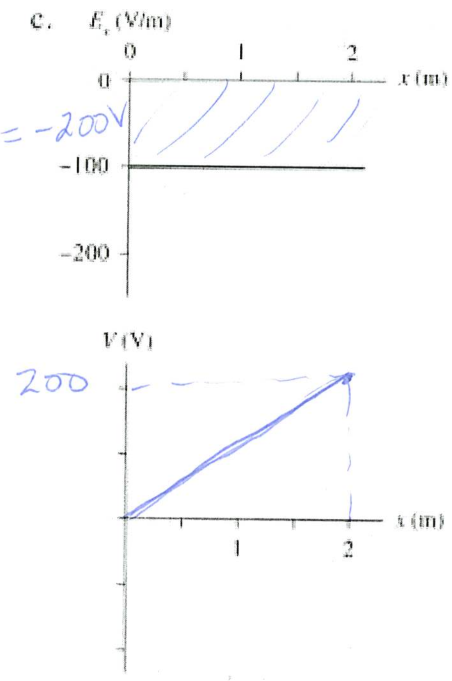


b.

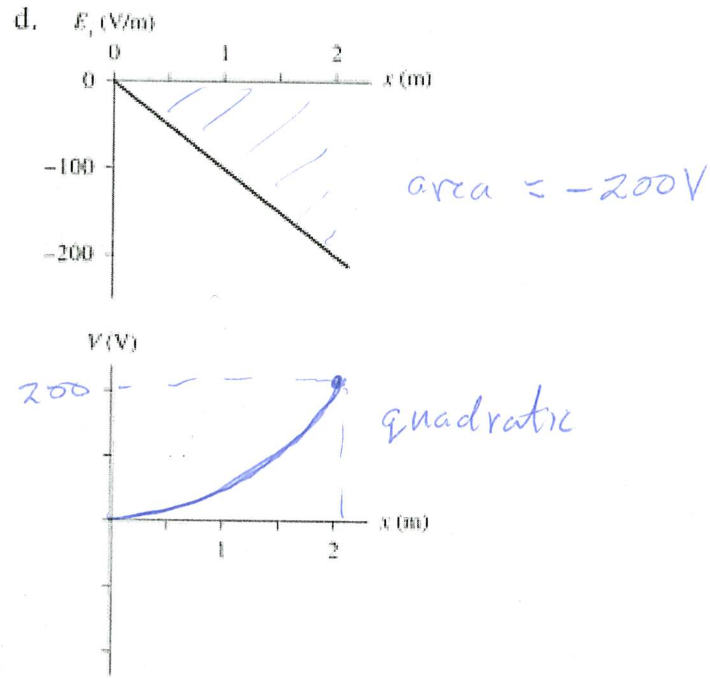


c.

area = −200V



d.



2.

For each contour map:

i. Estimate the electric fields $\vec{E}_a$ and $\vec{E}_b$ at points a and b. Don't forget that $\vec{E}$ is a vector. Show how you made your estimate.

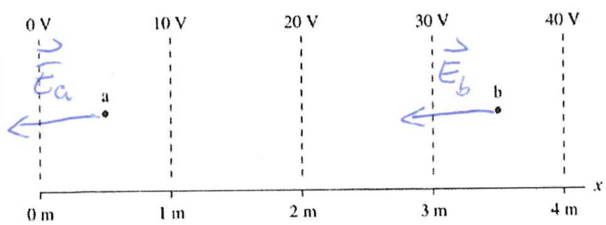
ii. On the contour map, draw the electric field vectors at points a and b.

$E_b = -\left(\frac{40V - 30V}{4m - 3m}\right) = -10\frac{V}{m}$

$E_x = -\frac{dV}{dx} \approx -\frac{\Delta V}{\Delta x}$

a.

$E_a = -\left(\frac{10V - 0V}{1m - 0m}\right) = -10\frac{V}{m}$

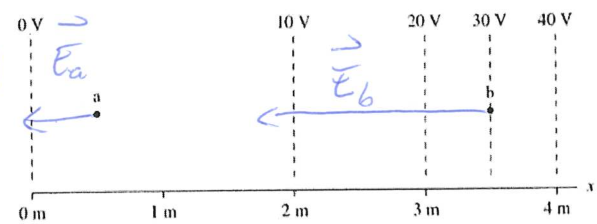


$\vec{E}_a =$ $-10\frac{V}{m}\,\hat{i}$

$\vec{E}_b =$ $-10\frac{V}{m}\,\hat{i}$

b.

$E_a = -\frac{(10V - 0V)}{2m - 1m} = -5\frac{V}{m}$



$\vec{E}_a =$ $-5\frac{V}{m}\,\hat{i}$

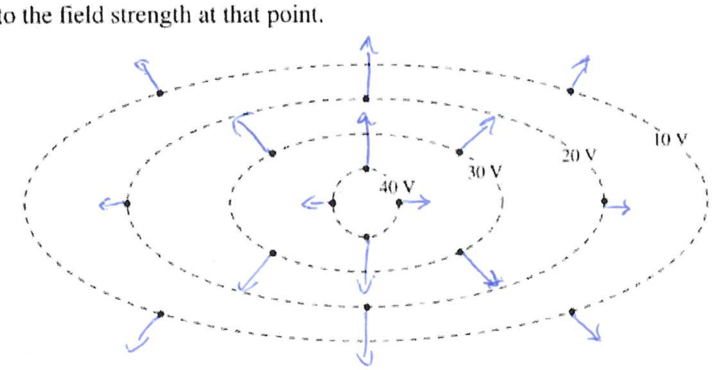
$\vec{E}_b =$ $-20\frac{V}{m}\,\hat{i}$

$E_b = -\frac{(40V - 20V)}{4m - 3m} = -20\frac{V}{m}$

3.

Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.

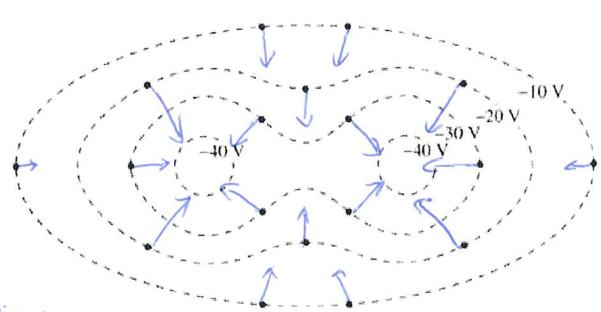
$\vec{E}$ pts from high pot. to low pot.



Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.

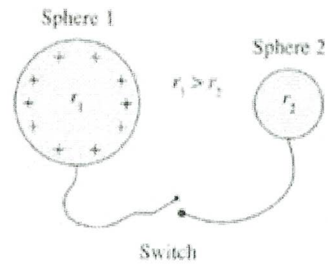
$\vec{E} \perp$ to equipotential lines

$\vec{E}$ large when equipotential spacing is small



4.

Two metal spheres are connected by a metal wire that has a switch in the middle. Initially the switch is open. Sphere 1, with the larger radius, is given a positive charge. Sphere 2, with the smaller radius, is neutral. Then the switch is closed. Afterward, sphere 1 has charge Q_1, is at potential V_1, and the electric field strength at its surface is E_1. The values for sphere 2 are Q_2, V_2, and E_2.



a. Is V_1 larger than, smaller than, or equal to V_2? Explain.

Equal. When switch close, sphere 1 + sphere 2 + wire act as a single conductor. In equil. the potential of a conductor is everywhere the same.

b. Is Q_1 larger than, smaller than, or equal to Q_2? Explain.

For a sphere $V = \frac{kQ}{r}$ $\therefore$ $\frac{kQ_1}{r_1} = \frac{kQ_2}{r_2}$ $\therefore$ $\frac{Q_1}{Q_2} = \frac{r_1}{r_2}$

$\therefore \boxed{Q_1 > Q_2.}$

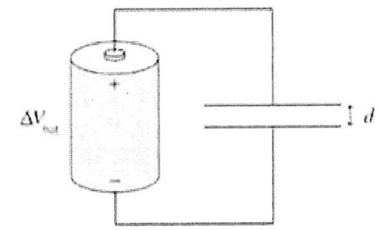
c. Is E_1 larger than, smaller than, or equal to E_2? Explain.

$E = \frac{kQ}{r^2}$ $\therefore$ $E_1 = \frac{kQ_1}{r_1^2}$ $E_2 = \frac{kQ_2}{r_2^2} = \frac{k}{r_2^2} Q_1 \frac{r_2}{r_1}$

$E_2 = \frac{kQ_1}{r_2 r_1}$

$\therefore \frac{E_1}{E_2} = \frac{\cancel{kQ_1}}{r_1^{\cancel{2}}} \frac{r_2 \cancel{r_1}}{\cancel{kQ_1}} = \frac{r_2}{r_1}$ $\therefore \boxed{E_1 < E_2}$

5. A parallel-plate capacitor with plate separation d is connected to a battery that has potential difference ΔV_{bat}. Without breaking any of the connections, insulating handles are used to increase the plate separation to $2d$.



a. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?

Battery is still connected.

$\therefore$ ΔV_c does not change. $\Delta V_c = \Delta V_{bat}$.

b. Does the capacitance change? If so, by what factor? If not, why not?

$C = \varepsilon_0 \frac{A}{d}$ $\therefore$ if $d \to 2d$ C decreases by a factor of 2.

c. Does the capacitor charge Q change? If so, by what factor? If not, why not?

$C = \frac{Q}{\Delta V_c}$ $\therefore$ $Q = C \Delta V_c$ since ΔV_c unchanged $C \to \frac{C}{2}$

$\boxed{Q \to \frac{Q}{2}}$

Q decreases by a factor of 2.

6. Each capacitor in the circuits below has capacitance C. What is the equivalent capacitance of the group of capacitors?

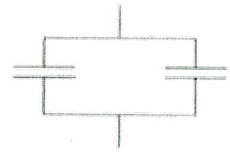
a.

$C_{eq} =$ $C/2$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

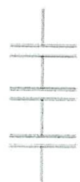
$\therefore C_{eq} = C/2$

b.



$C_{eq} =$ $2C$

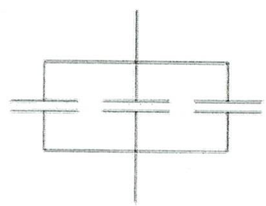
c.



$C_{eq} =$ $C/3$

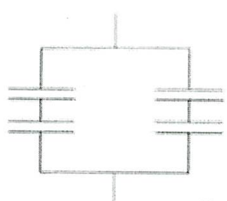
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

d.

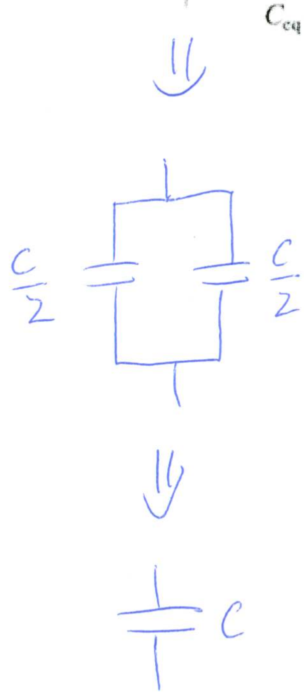


$C_{eq} =$ $3C$

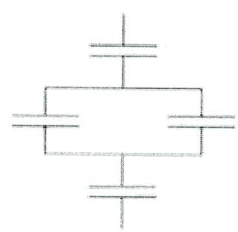
e.



$C_{eq} =$ C



f.



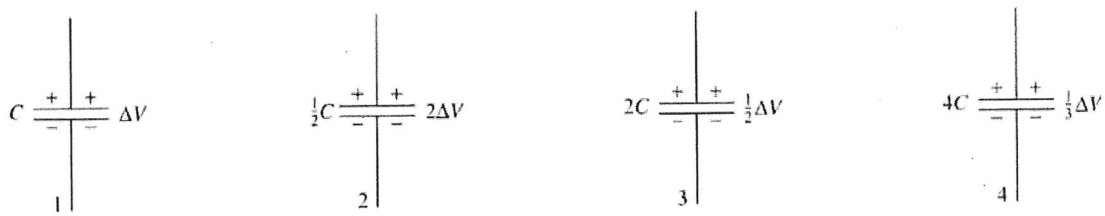
$C_{eq} =$ $\frac{2}{5}C$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{C} = \frac{2}{C} + \frac{1}{2C} = \frac{4}{2C} + \frac{1}{2C} = \frac{5}{2C}$$

$\therefore C_{eq} = \frac{2}{5}C$

7.

Rank in order, from largest to smallest, the energies $(U_C)_1$ to $(U_C)_4$ stored in each of these capacitors.



Order:

Explanation:

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 \qquad C = \frac{Q}{\Delta V}$$

$$= \frac{1}{2} \frac{Q^2}{C}$$

We're given C & ΔV $\therefore$ work w/ $U = \frac{1}{2} C (\Delta V)^2$

1. $U_1 = \frac{1}{2} C (\Delta V)^2$ (2)

2. $U_2 = \frac{1}{2} \left(\frac{1}{2} C\right)(2\Delta V)^2 = C (\Delta V)^2$ (1) largest.

3. $U_3 = \frac{1}{2} (2C) \left(\frac{\Delta V}{2}\right)^2 = \frac{1}{4} C (\Delta V)^2$ (3)

4. $U_4 = \frac{1}{2} (4C) \left(\frac{\Delta V}{3}\right)^2 = \frac{2}{9} C (\Delta V)^2$ (4) smallest